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Higher-Order Effects Induced Optical Solitons in Fiber

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In this paper, we study the existence conditions of the soliton solutions induced by considering the higher-order effects such as the third-order dispersion (TOD), self-steepening (SS), and self-frequency shift arising from stimulated Raman scattering (SRS) simultaneously in optical soliton communication. Based on the Jacobian expansion method, we successfully obtain bright and dark solitons. The results shows that the resultant inclusion is in agreement with Mollenauer *et al*. [*Physical Review Letters* **45** (1980) 1095] when the SRS is not considered; while when the SRS is considered, the existence conditions of the higher-order effects induced bright and dark solitons are not only quite different from those of the group velocity dispersion (GVD)-induced and self-phase modulation (SPM)-induced solitons, but also different from those of the TOD- and SS-induced solitons discussed by Mollenauer *et al*. [*Physical Review Letters* **45** (1980) 1095].

KEY WORDS: optical soliton; Schrödinger equation; higher-order nonlinear effects.

1. INTRODUCTION

It is well known that optical solitons have been the objects of extensive theoretical and experimental studies during the last three decades due to their potential applications in long distance communication and all-optical ultrafast switching devices. The optical soliton in a dielectric fiber was firstly proposed by Hasegawa and Tappert (1973a,b) and verified experimentally by Mollenauer *et al.* (1980). In the picosecond regime, the propagation of optical pulse in monomode optical fiber is governed by the nonlinear Schrödinger (NLS) equation.

$$E_z = i(\alpha_1 E_{tt} + \alpha_2 |E|^2 E), \tag{1}$$

where E is the slowly varying envelope of the electric field, the subscripts z and t are the spatial and temporal partial derivatives and the terms related to the real constants α_1 and $\alpha_2 > 0$ are corresponding to the group velocity dispersion (GVD)

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and self-phase modulation (SPM) well known in the fiber, and it admits bright and dark soliton-type pulse propagation in anomalous and normal dispersion regimes, respectively (Hasegawa and Tappert, 1973a,b).

However, for large channel handling capacity and for high speed it is necessary to transmit solitary waves at a high bit rate of ultrashort pulses (when the pulses are shorter than 100 fs). So it is very important that all higher-order effects, such as third-order dispersion (TOD), nonlinear dispersion and self-frequency shift (SFS) arising from stimulated Raman scattering (SRS), should be considered in the propagation of femitosecond pulses. Usually, the high-order terms related to the TOD, SS, and SRS are all treated as the perturbations for both the bright and dark soliton solutions. However, as Lou has reported in Sen-yue (2001), for a nonlinear system, when a new small effect is considered, there may be three types of effects for the old exact solutions. (a) The old exact solutions are changed perturbatively. (b) The old exact solutions are totally destroyed. (c) Some new types of exact solutions may be induced.

In this paper, we further study the existence conditions of the soliton solutions induced by considering the higher-order effects such as the TOD, SS, and SFS arising from SRS simultaneously. The results show that the resultant inclusion is in agreement with Sen-yue (2001) when the SRS is not considered; while when the SRS is considered, the existence conditions of the higher-order effects induced bright and dark solitons are not only quite different from those of the GVD- and SPM-induced solitons, but also different from those of the TOD- and SS-induced solitons discussed by Sen-yue (2001).

Considering the TOD, SS, and SRS effects simultaneously, the NLS equation should be modified as

$$E_z = i(\alpha_1 E_{tt} + \alpha_2 |E|^2 E) + \alpha_3 E_{ttt} + \alpha_4 (|E|^2 E)_t + \alpha_5 E(|E|^2)_t, \tag{2}$$

where $\alpha_3, \alpha_4, \alpha_5$ are the real parameters related to TOD, SS, SRS, respectively, which was derived by Kodama and Hasegawa (1987).

In order to seek some exact soliton solutions of Equation (2), we use the Jacobian expansion method. Firstly, we make the transformation

$$E(z,t) = u(\xi)\exp(i\theta), \xi = kz + ct + \delta_1, \theta = pz + qt + \delta_2, \tag{3}$$

where k, c, δ_1, p, q, δ_2 are constants, while $u(\xi)$ can be the complex because we consider the intensity $|E(z,t)|^2$. Substituting Equation (3) into Equation (2) and separating the real and imaginary parts leads to

$$\begin{aligned} &ku' + 2\alpha_1 qcu' - \alpha_3 c^3 u''' + 3\alpha_3 q^2 cu' - 3\alpha_4 cu^2 u' - 2\alpha_5 cu^2 u' = 0, \\ &pu - \alpha_1 c^2 u'' + q^2 \alpha_1 u - \alpha_2 u^3 - 3\alpha_3 qc^2 u'' + \alpha_3 q^3 u - \alpha_4 qu^3 = 0. \end{aligned} \tag{4}$$

Assuming

$$u(\xi) = a_0 + \sum_{j=1}^{l} sn^{j-1}(\xi)[a_j sn(\xi) + b_j cn(\xi)], \tag{5}$$

where $sn(\xi) \equiv sn(\xi, m)$, $cn(\xi) \equiv cn(\xi, m)$. By leading order analysis, we get $l = 1$. Therefore, the solution of Equation (2) has the form

$$u(\xi) = a_0 + a_1 sn(\xi) + b_1 cn(\xi), \tag{6}$$

where the parameters $a_0, a_1, b_1, k, c, p, q$ can be easily determined by Equation (4). Here we discuss two cases.

Case A: The corresponding solution reads

$$E(z,t) = b_1 cn(kz + ct + \delta_1)\exp(pz + qt + \delta_2), \tag{7}$$

where

$$\begin{aligned}
b_1 &= \pm\sqrt{6}\sqrt{\frac{\alpha_3}{2\alpha_5 + 3\alpha_4}}mc,\\
q &= \frac{1}{6}\frac{-2\alpha_5\alpha_1 + 3\alpha_3\alpha_2 - 3\alpha_1\alpha_4}{\alpha_3(\alpha_4 + \alpha_5)},\\
k &= -\frac{1}{12}\big(\big(-24\alpha_3^2c^2m^2\alpha_4^2 - 3\alpha_1^2\alpha_4^2 + 12\alpha_3^2c^2\alpha_4^2 - 8\alpha_5\alpha_1^2\alpha_4 + 24\alpha_3^2c^2\alpha_5\alpha_4\\
&\quad - 48\alpha_3^2c^2m^2\alpha_5\alpha_4 - 6\alpha_1\alpha_3\alpha_2\alpha_4 - 24\alpha_3^2c^2m^2\alpha_5^2 - 4\alpha_5^2\alpha_1^2 + 9\alpha_3^2\alpha_2^2\\
&\quad + 12\alpha_3^2c^2\alpha_5^2\big)c\big)/(\alpha_3(\alpha_4 + \alpha_5)^2),\\
p &= -\frac{1}{216}\big(27\alpha_1^3\alpha_4^3 - 108\alpha_3^2c^2\alpha_1\alpha_4^3 + 216\alpha_4^3c^2\alpha_3^2m^2\alpha_1 + 72\alpha_5\alpha_1^3\alpha_4^2\\
&\quad + 324\alpha_3^3c^2\alpha_2\alpha_4^2 - 648\alpha_4^2\alpha_2\alpha_3^3m^2c^2 + 432\alpha_4^2c^2\alpha_3^2m^2\alpha_5\alpha_1 - 27\alpha_3\alpha_2\alpha_1^2\alpha_4^2\\
&\quad - 216\alpha_3^2c^2\alpha_5\alpha_1\alpha_4^2 - 72\alpha_5\alpha_1^2\alpha_3\alpha_2\alpha_4 + 216\alpha_4c^2\alpha_3^2m^2\alpha_5^2\alpha_1\\
&\quad + 648\alpha_3^3c^2\alpha_2\alpha_5\alpha_4 + 60\alpha_5^2\alpha_1^3\alpha_4 - 27\alpha_3^2\alpha_2^2\alpha_1\alpha_4 - 108\alpha_3^2c^2\alpha_5^2\alpha_1\alpha_4\\
&\quad - 1296\alpha_4\alpha_2\alpha_3^3m^2c^2\alpha_5 - 648\alpha_2\alpha_3^3m^2c^2\alpha_5^2 + 16\alpha_5^3\alpha_1^3 - 36\alpha_5^2\alpha_1^2\alpha_3\alpha_2\\
&\quad + 27\alpha_3^3\alpha_2^3 + 324\alpha_3^3c^2\alpha_2\alpha_5^2\big)/\big(\alpha_3^2(\alpha_4 + \alpha_5)^3\big),
\end{aligned} \tag{8}$$

while c is an arbitrary constant. When modulous number $m \to 1$, the corresponding solution becomes a bright soliton solution with the form

$$E(z,t) = b_1 \operatorname{sech}(kz + ct + \delta_1)\exp(pz + qt + \delta_2). \tag{9}$$

Case B: The corresponding solution reads

$$E(z,t) = a_1 sn(kz + ct + \delta_1)\exp(pz + qt + \delta_2). \tag{10}$$

where

$$
\begin{aligned}
q &= \frac{1}{6}\frac{-2\alpha_5\alpha_1 + 3\alpha_3\alpha_2 - 3\alpha_1\alpha_4}{\alpha_3(\alpha_4+\alpha_5)},\\
a_1 &= \pm\sqrt{-\frac{6\alpha_3}{2\alpha_5+3\alpha_4}mc},\\
k &= -\frac{1}{12}c\big(12\alpha_3^2c^2m^2\alpha_4^2 + 24\alpha_3^2c^2m^2\alpha_5\alpha_4 + 12\alpha_3^2c^2m^2\alpha_5^2 - 8\alpha_5\alpha_1^2\alpha_4\\
&\quad - 4\alpha_5^2\alpha_1^2 - 6\alpha_1\alpha_3\alpha_2\alpha_4 - 3\alpha_1^2\alpha_4^2 + 12\alpha_3^2c^2\alpha_4^2\\
&\quad + 24\alpha_3^2c^2\alpha_5\alpha_4 + 12\alpha_3^2c^2\alpha_5^2 + 9\alpha_3^2\alpha_2^2\big)/(\alpha_3(\alpha_4+\alpha_5)^2),\\
p &= -\frac{1}{216}\big(27\alpha_3^3\alpha_2^3 - 27\alpha_3\alpha_2\alpha_1^2\alpha_4^2 - 36\alpha_5^2\alpha_1^2\alpha_3\alpha_2 + 16\alpha_5^3\alpha_1^3\\
&\quad - 72\alpha_5\alpha_1^2\alpha_3\alpha_2\alpha_4 - 27\alpha_3^2\alpha_2^2\alpha_1\alpha_4 + 60\alpha_5^2\alpha_1^3\alpha_4 + 72\alpha_5\alpha_1^3\alpha_4^2\\
&\quad - 216\alpha_3^2c^2\alpha_5\alpha_1\alpha_4^2 - 108\alpha_3^2c^2\alpha_5^2\alpha_1\alpha_4 + 648\alpha_3^3c^2\alpha_2\alpha_5\alpha_4 + 324\alpha_3^3c^2\alpha_2\alpha_4^2\\
&\quad + 324\alpha_3^3c^2\alpha_2\alpha_5^2 - 108\alpha_3^2c^2\alpha_1\alpha_4^3 + 27\alpha_1^3\alpha_4^3 - 108\alpha_4c^2\alpha_3^2m^2\alpha_5^2\alpha_1\\
&\quad + 648\alpha_4\alpha_2\alpha_3^3m^2c^2\alpha_5 + 324\alpha_4^2\alpha_2\alpha_3^3m^2c^2 - 216\alpha_4^2c^2\alpha_3^2m^2\alpha_5\alpha_1\\
&\quad - 108\alpha_4^3c^2\alpha_3^2m^2\alpha_1 + 324\alpha_2\alpha_3^3m^2c^2\alpha_5^2\big)/(\alpha^3(\alpha_4+\alpha_5)^3),
\end{aligned} \tag{11}
$$

while c is also an arbitrary constant. When modulous number $m \to 1$, the corresponding solution is a dark soliton solution with the form

$$E(z,t) = a_1 \tanh(kz + ct + \delta_1)\exp(pz + qt + \delta_2). \tag{12}$$

From the expressions of the bright and dark solitons with Equations (9) and (12), we can easily see that if the SRS effect is neglected, i.e., $\alpha_5 = 0$, the bright soliton exists only for

$$\alpha_3\alpha_4 > 0, \tag{13}$$

and the dark soliton exists only for

$$\alpha_3\alpha_4 < 0. \tag{14}$$

When the TOD is neglected ($\alpha_3 \to 0$), both the bright and dark solitons disappear ($|E|^2 \to 0$). If the SS is not considered ($\alpha_4 \to 0$), both the bright and dark solitons are destroyed ($|E|^2 \to \infty$). The balance between the TOD and the SS (α_3 and α_4 are all finite) leads to a localized finite bright soliton (for $\alpha_3\alpha_4 > 0$) and dark soliton (for $\alpha_3\alpha_4 < 0$). For the GVD- and SPM-induced solitons the bright soliton exist only for the fiber with the anomalous dispersion ($\alpha_1 > 0$) and the dark soliton exist only for the fiber with the normal dispersion ($\alpha_1 < 0$) (Hasegawa and Tappert, 1973a,b; Zakharov *et al.*, 1980, 1984; Cai *et al.*, 1997). However, from the expressions Equations (9) and (12), the bright and dark solitons are valid for $\alpha_1 \to 0$ and $\alpha_2 \to 0$. The resultant conclusions are in agreement with the one reported by Sen-yue (2001).

If the SRS effect is considered, i.e., $\alpha_5 \neq 0$. When the TOD is neglected ($\alpha_3 \to 0$), the bright and dark solitons disappear ($|E|^2 \to 0$). It is worth noting that if the SS is not considered ($\alpha_4 \to 0$), the bright and dark solitons are not destroyed, which is different from the result of the case when $\alpha_5 = 0$. So from the phenomenon we can see that, the SRS played an important role in inducing new types of soliton solutions. From Equations (9) and (12), one can easily see that these solitons are induced by the TOD, SS, and SRS. And there may exist a balance of the cooperation of all these effects: GVD, SPM, TOD, SS, SRS, which lead to a stable pulse propagation. One can also see that the higher-order effects induced bright and dark solitons exist even if there is no effect of the GVD and SPM, i.e., the bright and dark solitons Equations (9) and (12) are valid for $\alpha_1 \to 0$ and $\alpha_2 \to 0$. However, the higher-order effects induced bright soliton may exist both for the anomalous-dispersion fiber and for the normal-dispersion fiber if $\frac{\alpha_3}{2\alpha_5+3\alpha_4} > 0$ and the higher-order effects induced dark soliton may also exist in anomalous-dispersion fiber and the normal-dispersion fiber if $\frac{\alpha_3}{2\alpha_5+3\alpha_4} < 0$.

To conclusion, based on the Jacobian expansion method to the nonlinear Schrödinger equation with the TOD, SS, and SRS effects, we obtain that the SRS effect play an important role in inducing new types of soliton solutions. When the SRS is considered, the existence conditions of the higher-order effects induced bright and dark solitons are not only quite different from those of the GVD- and SPM-induced solitons, but also different from those of the TOD- and SS-induced solitons.

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